EngineeringNotes.net

Heat Transfer Key Equations

Here's a hot nomenclature

q	**Power** (heat transfer per unit time), measured in W
q''	**Heat flux** (power per unit area), measured in $W\ m^{-2}$
Q	**Heat** (thermal energy transferred over a time period), measured in J
k	**Thermal conductivity** (a material property), measured in $W\ m^{-1}K^{-1}$
h	**Convective heat transfer coefficient**, measured in $W\ m^2K^{-2}$
α	**Thermal diffusivity**, measured in m^2s^{-1}
R_t	**Thermal resistance**, measured in $K\ W^{-1}$
c	**Specific heat**, measured in $J\ kg^{-1}K^{-1}$
L_C	**Characteristic length**, measured in m

Conduction

Fourier's Law $$q'' = -k\frac{dt}{dx}$$

Linear temperature distribution $$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

Convection

Newton's Cooling Law $$q'' = h(T_S - T_\infty)$$

Radiation

Stefan-Boltzmann's Law $$q'' = E_b = \sigma T^4$$

Emissivity $$\varepsilon = \frac{E}{E_b}$$

E_B is the emissive power, and σ is the Stefan-Boltzmann constant, $5.67 \times 10^{-8}\ W\ m^{-2}K^{-4}$

Heat Diffusion Equation

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}_g = \rho c\frac{\partial T}{\partial t}$$

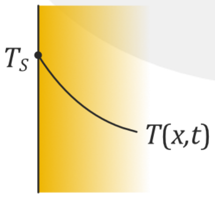
Steady, 1D, no $\dot{q}_g$ $$T(x) = c_1 x$$

Energy balance of a control volume $$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

Heat Diffusion in 1 dimension $$k\frac{\partial^2 T}{\partial x^2} + \dot{q}_g = \rho c\frac{\partial T}{\partial t}$$

Steady, 1D, with $\dot{q}_g$ $$T(x) = \frac{\dot{q}x^2}{2k} + c_1 x + c_2$$

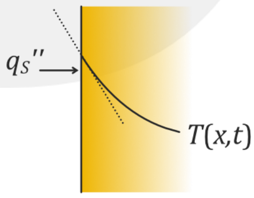
Thermal diffusivity, α $$\alpha = \frac{k}{\rho c}$$



Dirichlet Condition

Constant surface temperature

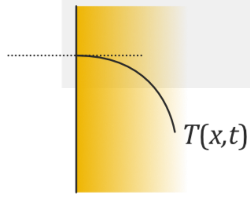
$T(0,t) = T_s$



Von

Constant heat flux

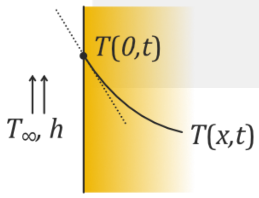
$$-k\frac{\partial T}{\partial x_0} = q_s$$



Neumann

Insulated surface

$$\frac{\partial T}{\partial x_0} = 0$$



Convection

Combination

$$-k\frac{\partial T}{\partial x_0} = h[T_\infty - T(0,t)]$$

Thermal Resistance

Thermal resistance $R_t = \frac{\Delta T}{q}$

Per unit area $R_t'' = \frac{\Delta T}{q''}$

For Conduction $R_{t\ cond} = \frac{L}{kA}$

For Convection $R_{t\ conv} = \frac{1}{hA}$

Total Heat Transfer Coefficient

$q = UA\Delta T$ $R_{total} = \frac{1}{UA}$

Lumped (Lang) Capacitance

This assumption is valid when $Bi < 0.1$

$$\rho c V \frac{dT}{dt} = -hA(T - T_\infty)$$

Temperature-time $\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho c V}t}$

Time constant $\tau_t = \frac{\rho c V}{hA}$

Non-Dimensional Analysis

For when $Bi > 0.1$

$$x^* = \frac{x}{L_C} \quad \theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} \quad t^* = Fo = \frac{\alpha t}{L_C^2}$$

$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$ becomes $\frac{\partial^2 \theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial Fo}$

Which solves with tabulated constants:

$$\theta^* \approx C_1 e^{-\zeta_1^2 Fo}\cos(\zeta_1 x^*)$$

Fins

$$\frac{d}{dx}\left(\frac{A_c dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

For uniform cross-section $\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0$

Parametrising $m^2 = \frac{hP}{kA_C}$ $\theta = T - T_\infty$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{mx} - C_2 e^{-mx}$$

Tip Conditions

Convection $-k\frac{d\theta}{dx_L} = h\theta(L)$

Adiabatic $\frac{d\theta}{dx_L} = 0$

Fixed temperature $\theta(L) = \theta_L$

Infinite length ($mL > 2.65$) $\theta(L) = 0$

Important Quantities

Biot Number $Bi = \frac{hL_C}{k}$

$$Bi = \frac{R_{cond}}{R_{conv}} = \frac{\Delta T_{solid}}{\Delta T_{fluid}}$$

Characteristic Length $L_C^I = \frac{V}{A}$

$L_C^{II} = x\ of\ \max \Delta T$

Fourier Number $Fo = \frac{\alpha t}{L_C^2}$