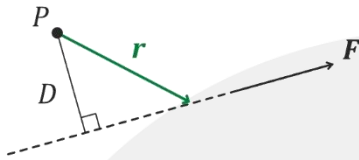


Mechanics Key Equations

Forces & Moments as Vectors



The magnitude of a moment about a point P in scalar form:

$$|M_P| = |F|D$$

In vector form, this is given as the cross product of the position and force vectors:

$$\mathbf{M}_P = \mathbf{r} \times \mathbf{F} \quad |M_P| = |\mathbf{r} \times \mathbf{F}|$$

Rectilinear Kinematics of Particles

The SUVAT equations for constant, linear acceleration:

$$v = u + at$$

$$v^2 = u^2 + 2as$$

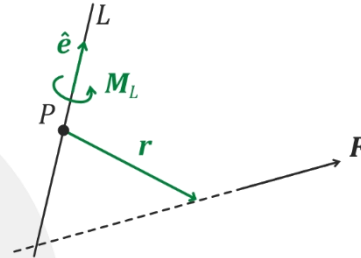
$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

Integral relations:

$$s \xrightarrow{\frac{ds}{dt}} v \xrightarrow{\frac{dv}{dt}} a$$



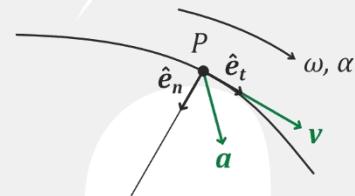
Moments about a line:

$$\mathbf{M}_L = (\hat{e} \cdot \mathbf{r} \times \mathbf{F}) \hat{e}$$

$$\mathbf{M}_L = (\hat{e} \cdot \mathbf{M}_P) \hat{e}$$

In both cases, the direction and sign of the moments are given by the right-hand rule.

Curvilinear Kinematics of Particles



Angular velocity & acceleration:

$$\omega = \dot{\theta} = \frac{d\theta}{dt}$$

$$\alpha = \dot{\omega} = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$$

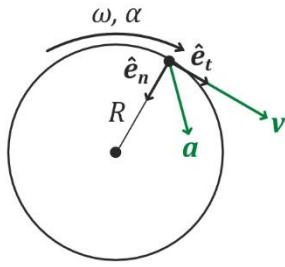
$$\text{Unit conversion: } \text{rad/s} = \text{RPM} \times \frac{\pi}{30}$$

Velocity is tangential:

$$\mathbf{v} = r\omega \quad \mathbf{v} = v \hat{e}_t = r\omega \hat{e}_t$$

Acceleration has both components:

$$\mathbf{a} = r\alpha \quad \mathbf{a} = a \hat{e}_t + v\omega \hat{e}_n = a \hat{e}_t + \frac{v^2}{r} \hat{e}_n$$



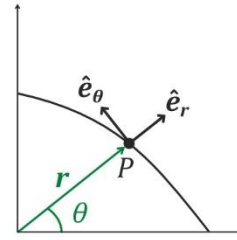
For Circular motion:

$$v = R\omega$$

$$\mathbf{v} = R\omega \hat{\mathbf{e}}_t$$

$$a = R\alpha$$

$$\mathbf{a} = R\alpha \hat{\mathbf{e}}_t + \frac{v^2}{R} \hat{\mathbf{e}}_n$$



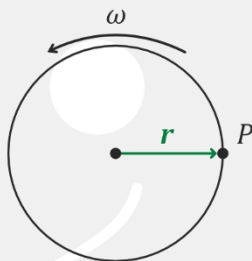
In Polar Coordinates:

$$\mathbf{v} = \dot{r} \hat{\mathbf{e}}_r + r\dot{\theta} \hat{\mathbf{e}}_\theta$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2) \hat{\mathbf{e}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{\mathbf{e}}_\theta$$

Where $\mathbf{r} = r\hat{\mathbf{e}}_r$

Kinematics of Rigid Bodies



Velocity of a point P on a rigid body:

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}$$

Relative motion:

$$\mathbf{v}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

where $\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$

For a sliding contact at point A:

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B} + \mathbf{v}_r$$

Where \mathbf{v}_r is the velocity relative to the slot.

Friction

F is the frictional force, R the normal reaction with the surface:

$$F = \mu R$$

$$\mu_s = \tan \alpha_{max}$$

Acceleration of a point on a rigid body:

$$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

where $\boldsymbol{\alpha} \times \mathbf{r}$ is the tangential component and $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ is the normal component.

For general plane motion, the normal component becomes:

$$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} - \omega^2 \mathbf{r}$$

Relative motion:

$$\mathbf{a}_{A/B} = \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

where $\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$

For a sliding contact at point A:

$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B} + \mathbf{a}_r + 2\boldsymbol{\omega} \times \mathbf{v}_r$$

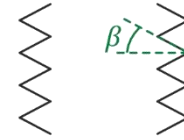
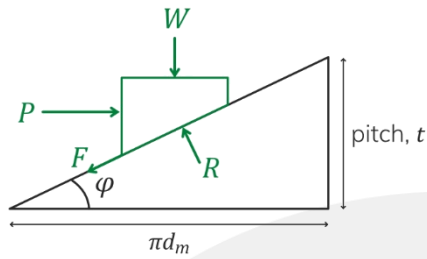
Where \mathbf{a}_r is the acceleration relative to the slot.

μ_s is the static coefficient of friction ($\sim 20\% > \mu_k$, the kinetic coefficient of friction).

α_{max} is the friction angle – the maximum angle of inclination before the object slips:

Screw threads:

$$\tan\phi = \frac{t}{\pi d_m}$$



Square Screw Threads:

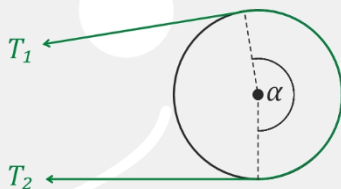
$$P = \left(\frac{\tan\phi + \mu_s}{1 - \mu_s \tan\phi} \right) \times W$$

For v-threads:

$$P = \left(\frac{\tan\phi + \mu_s \sec\beta}{1 - \mu_s \sec\beta \tan\phi} \right) \times W$$

Belts:

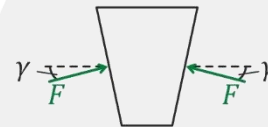
For square belts:



$$\frac{T_1}{T_2} = e^{\mu_s \alpha}$$

Where μ_s is the static coefficient of friction, and α the wrap angle.

For v-shaped belts:



$$\frac{T_1}{T_2} = e^{\frac{\mu_s \alpha}{\sin\gamma}}$$

Where γ is the angle of the v-section.

Clutches

For uniform pressure:

$$F = \pi p (R_1^2 - R_2^2)$$

$$T = \frac{2}{3} \mu F \left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2} \right)$$

For uniform wear:

$$T = \mu F \left(\frac{R_1 + R_2}{2} \right)$$