Structures Key Equations

Statical Determinacy

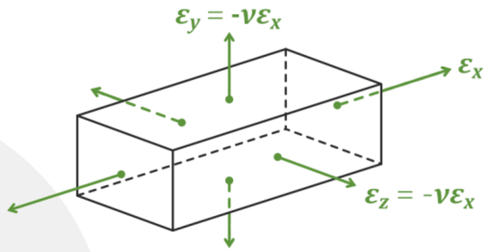
Mechanism	$b + n < 2j$
May be determinate	$b + n = 2j$
Indeterminate	$b + n > 2j$

- b is the number of bars
- n is the number of reactions
- j the number of pins

Stress & Strain

Engineering Stress	$\sigma = \frac{F}{A}$
Engineering Strain	$\varepsilon = \frac{x}{L}$
Youngs Modulus	$E = \frac{\sigma}{\varepsilon}$
Poisson's Ratio	$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$
Hydrostatic Stress	$\sigma_H = \sigma_x = \sigma_y = \sigma_z$

Rectangular Cross-Sections



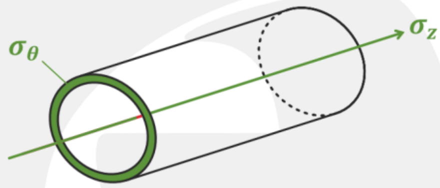
Change in Volume	$\Delta V = \varepsilon_V \times V$
Volumetric Strain	$\varepsilon_V = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$
Bulk Modulus	$K = \frac{\sigma_H}{\varepsilon_V}$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right) + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta T$$

Cylindrical Thin-Walled Pressure Vessels



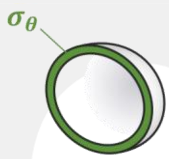
Hoop Stress, σ_θ	$\sigma_\theta = \frac{PR_i}{t}$
Axial Stress, σ_z	$\sigma_z = \frac{\sigma_\theta}{2} = \frac{PR_i}{2t}$
Axial strain, ε_z	$\varepsilon_z = \frac{\Delta L}{L}$
Hoop strain, ε_θ	$\varepsilon_\theta = \frac{\Delta r}{r}$
Volumetric Strain	$\varepsilon_V = \frac{\Delta V}{V} = 2\varepsilon_\theta + \varepsilon_z$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_\theta + \sigma_r\right)\right) + \alpha\Delta T$$

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right) + \alpha\Delta T$$

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right)\right) + \alpha\Delta T$$

Spherical Thin-Walled Pressure Vessels



Hoop Stresses, σ_θ, σ_ϕ	$\sigma_\theta = \sigma_\phi = \frac{PR_i}{2t}$
Hoop Strains, ε_θ, σ_ϕ	$\varepsilon_\theta = \varepsilon_\phi = \frac{\Delta r}{r}$
Volumetric Strain	$\varepsilon_V = \frac{\Delta V}{V} = 3\varepsilon_\theta$

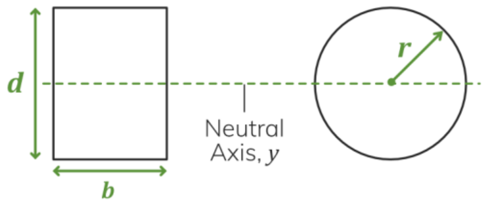
$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_\phi\right)\right) + \alpha\Delta T$$

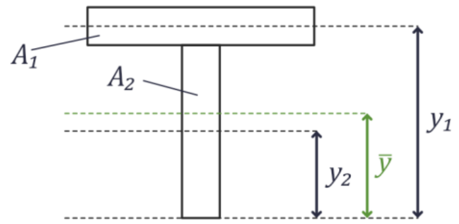
$$\varepsilon_\phi = \frac{1}{E}\left(\sigma_\phi - \nu\left(\sigma_r + \sigma_\theta\right)\right) + \alpha\Delta T$$

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\left(\sigma_\theta + \sigma_\phi\right)\right) + \alpha\Delta T$$

- Take radial stresses and strains as zero in both spheres and cylinders.
- Axial strain in a sphere is also zero.

Second Moment of Area





$$I = \frac{bd^3}{12}$$

$$I = \frac{\pi r^4}{4}$$

$$\bar{y}(A_{total}) = y_1(A_1) + y_2(A_2)$$

$$I_{xx} = [I_1 + A_1(y_1 - \bar{y})^2] + [I_2 + A_2(\bar{y} - y_2)^2]$$

Beam Theory

Key Relation $$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Radius & Moment $$R = \frac{d^2v}{dx^2} = \frac{1}{EI}M$$

Slope Angle $$\frac{dv}{dx} = \frac{1}{EI}\int M\,dx$$

Deflection $$v = \frac{1}{EI}\iint M\;dx$$

Torsion

Key Relation $$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

Shear Stress & Strain $$\gamma = \frac{\tau}{G}$$

Shear Modulus $$G = \frac{E}{2(1+v)}$$

Torsional Stiffness, K_T $$K_T = \frac{T}{\theta} = \frac{JG}{L}$$

$$\frac{1}{K_{T,total}} = \frac{1}{K_{T1}} + \frac{1}{K_{T2}} \ldots$$

Torsion in Thin-Walled Shafts

Shear Strain $$\gamma = \frac{GR_0}{L}$$

Shear Stress $$\tau = \frac{GR_0\theta}{L}$$

$$\tau = \frac{T}{2\pi R_0^2 t}$$

Torsion in Solid & Hollow Shafts

Shear Strain $$\gamma(r) = \frac{r\theta}{L}$$

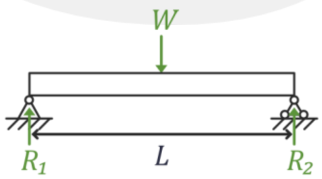
Shear Stress $$\tau = \frac{Gr\theta}{L}$$

Torque $$T = \frac{G\theta J}{L}$$

2nd Polar Moment of Area $$J = \frac{\pi D^4}{32}$$

Standard Slope & Deflection of Beams

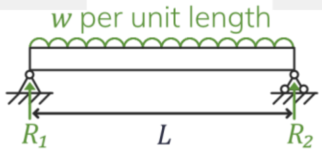
Simply Supported with Point Mass



End Slope $$\frac{WL^2}{16EI}$$

Central Deflection $$\frac{WL^3}{48EI}$$

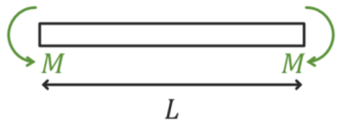
Simply Supported with Distributed Load



End Slope $$\frac{wL^3}{24EI}$$

Central Deflection $$\frac{5wL^4}{384EI}$$

Moment Supported



End Slope

$$\frac{ML}{2EI}$$

Central Deflection

$$\frac{ML^2}{8EI}$$

Cantilever with Moment

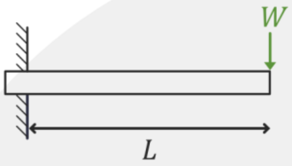
End Slope

$$\frac{ML}{EI}$$

End Deflection

$$\frac{ML^2}{2EI}$$

Cantilever with Point Load



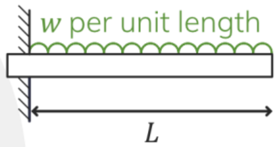
End Slope

$$\frac{WL^2}{2EI}$$

End Deflection

$$\frac{WL^3}{3EI}$$

Cantilever with Distributed Load



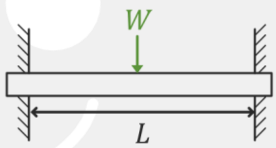
End Slope

$$\frac{wL^3}{6EI}$$

End Deflection

$$\frac{wL^4}{8EI}$$

Built in at Both Ends with Central Point Load



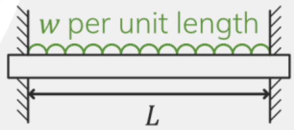
End Moment

$$\frac{WL}{8}$$

Central Deflection

$$\frac{WL^3}{192EI}$$

Built in at Both Ends with Distributed Load



End Moment

$$\frac{wL^2}{12}$$

Central Deflection

$$\frac{wL^4}{384EI}$$

Principal Stresses

Direct stresses

$$S_N = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Principal stresses

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2}$$

Shear stresses

$$S_S = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

Direct direction

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Shear direction

$$\tan 2\theta_S = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

- θ_P and θ_S are always 45° apart

Maximum Shear Stresses, $\hat{\tau}$

One of σ_1, σ_2 positive, one negative $\quad \hat{\tau} = \frac{\sigma_1 - \sigma_2}{2}$

Both σ_1, σ_2 positive $\quad \hat{\tau} = \frac{\sigma_1}{2}$

Both σ_1, σ_2 positive $\quad \hat{\tau} = \frac{\sigma_2}{2}$

Tresca Failure Criterion

$$\hat{\tau} = \max\left(\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_1 - \sigma_3}{2}\right|\right)$$

Von Mises (Strain Energy) Failure Criterion

$$\hat{\tau} = \sqrt{\frac{1}{3}\left[\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \left(\frac{\sigma_2 - \sigma_3}{2}\right)^2 + \left(\frac{\sigma_3 - \sigma_1}{2}\right)^2\right]}$$

Principal Strains

Direct strains

$$e_n = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Principal Strains

$$e_{1,2} = \frac{e_x + e_y}{2} \pm \frac{1}{2}\sqrt{\left(e_x - e_y\right)^2 + \gamma_{xy}^2}$$

Shear strains

$$\frac{e_s}{2} = \frac{e_x - e_y}{2}\sin 2\theta - \frac{\gamma_{xy}}{2}\cos 2\theta$$

Direct strain direction

$$\tan 2\theta_P = \frac{\gamma_{xy}}{e_x - e_y}$$

Shear direction

Type equation here.

Maximum Shear Strains

$$e_{s\,max,min} = \pm\frac{e_1 - e_2}{2}$$

Finding σ_x from shear stresses and strains

$$\sigma_x = \frac{E\left(e_x + \nu e_y\right)}{1 - \nu^2}$$