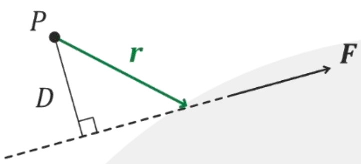
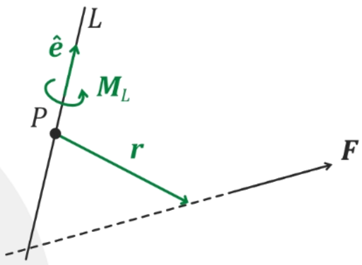
EngineeringNotes.net

Mechanics & Dynamics Key Equations

Forces & Moments as Vectors





The magnitude of a moment about a point P in scalar form:

$$|\boldsymbol{M}_P| = |\boldsymbol{F}|D$$

In vector form, this is given as the cross product of the position and force vectors:

$$\boldsymbol{M}_P = \boldsymbol{r} \times \boldsymbol{F} \qquad |\boldsymbol{M}_P| = |\boldsymbol{r} \times \boldsymbol{F}|$$

Moments about a line:

$$\boldsymbol{M}_L = (\hat{\boldsymbol{e}} \,.\, \boldsymbol{r} \times \boldsymbol{F})\, \hat{\boldsymbol{e}}$$

$$\boldsymbol{M}_L = (\hat{\boldsymbol{e}} \,.\, \boldsymbol{M}_P)\, \hat{\boldsymbol{e}}$$

In both cases, the direction and sign of the moments are given by the right-hand rule.

Rectilinear Kinematics of Particles

The SUVAT equations for constant, linear acceleration:

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

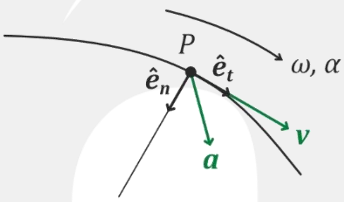
$$s = vt - \frac{1}{2}at^s$$

$$s = \frac{1}{2}(u + v)t$$

Integral relations:

$$s \xrightarrow{\frac{ds}{dt}} v \xrightarrow{\frac{dv}{dt}} a$$

Curvilinear Kinematics of Particles



Angular velocity & acceleration:

$$\omega = \dot{\theta} = \frac{d\theta}{dt}$$

$$\alpha = \ddot{\theta} = \frac{d^2\theta}{dt} = \frac{d\omega}{dt}$$

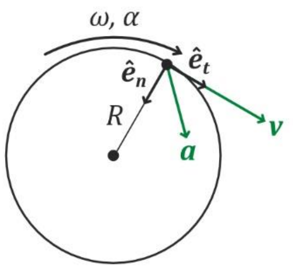
Unit conversion: $rad/s \; = RPM \times \frac{\pi}{30}$

Velocity is tangential:

$$v = r\omega \qquad \boldsymbol{v} = v\, \hat{\boldsymbol{e}}_t = r\omega\, \hat{\boldsymbol{e}}_t$$

Acceleration has both components:

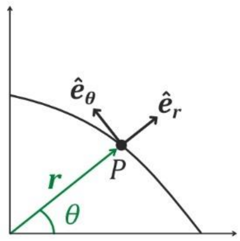
$$a = r\alpha \qquad \boldsymbol{a} = a\, \hat{\boldsymbol{e}}_t + v\omega\, \hat{\boldsymbol{e}}_n = a\, \hat{\boldsymbol{e}}_t + \frac{v^2}{r}\, \hat{\boldsymbol{e}}_n$$



For Circular motion:

$v = R\omega$ $\quad\quad$ $\boldsymbol{v} = R\omega\,\hat{\boldsymbol{e}}_t$

$a = R\alpha$ $\quad\quad$ $\boldsymbol{a} = R\alpha\,\hat{\boldsymbol{e}}_t + \frac{v^2}{R}\hat{\boldsymbol{e}}_n$



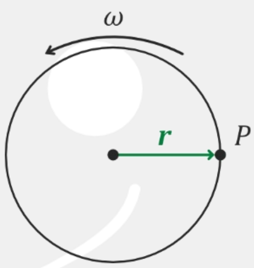
In Polar Coordinates:

$\boldsymbol{v} = \dot{r}\,\hat{\boldsymbol{e}}_r + r\dot{\theta}\,\hat{\boldsymbol{e}}_\theta$

$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\boldsymbol{e}}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{e}}_\theta$

Where $\boldsymbol{r} = r\hat{\boldsymbol{e}}_r$

Kinematics of Rigid Bodies



Velocity of a point P on a rigid body:

$\boldsymbol{v}_P = \boldsymbol{\omega} \times \boldsymbol{r}$

Relative motion:

$\boldsymbol{v}_{A/B} = \boldsymbol{\omega} \times \boldsymbol{r}_{A/B}$

where $\boldsymbol{a}_{A/B} = \boldsymbol{a}_A - \boldsymbol{a}_B$

For a sliding contact at point A:

$\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{\omega} \times \boldsymbol{r}_{A/B} + \boldsymbol{v}_r$

Where $\boldsymbol{v}_r$ is the velocity relative to the slot.

Acceleration of a point on a rigid body:

$\boldsymbol{a} = \boldsymbol{\alpha} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$

where $\boldsymbol{\alpha} \times \boldsymbol{r}$ is the tangential component and $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$ is the normal component.

For general plane motion, the normal component becomes:

$\boldsymbol{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2\boldsymbol{r}$

Relative motion:

$\boldsymbol{a}_{A/B} = \boldsymbol{\alpha} \times \boldsymbol{r}_{A/B} - \omega^2\boldsymbol{r}_{A/B}$
where $\boldsymbol{a}_{A/B} = \boldsymbol{a}_A - \boldsymbol{a}_B$

For a sliding contact at point A:

$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{\alpha} \times \boldsymbol{r}_{A/B} - \omega^2\boldsymbol{r}_{A/B} + \boldsymbol{a}_r + 2\boldsymbol{\omega} \times \boldsymbol{v}_r$

Where $\boldsymbol{a}_r$ is the acceleration relative to the slot.

Friction

F is the frictional force, R the normal reaction with the surface:

$F = \mu R$

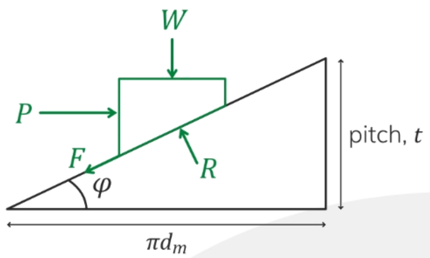
$\mu_s = \tan\alpha_{max}$

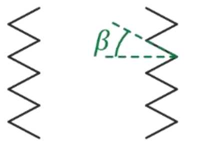
μ_s is the static coefficient of friction (~20% > μ_k, the kinetic coefficient of friction).

α_{max} is the friction angle – the maximum angle of inclination before the object slips:

Screw threads:

$$tan\phi = \frac{t}{\pi d_m}$$





Square Screw Threads:

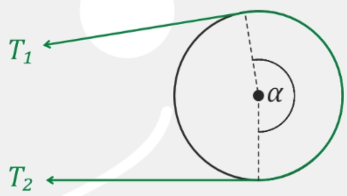
$$P = \left(\frac{tan\phi + \mu_s}{1 - \mu_s tan\phi}\right) \times W$$

For v-threads:

$$P = \left(\frac{tan\phi + \mu_s\, sec\beta}{1 - \mu_s\, sec\beta\, tan\phi}\right) \times W$$

Belts:

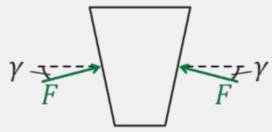
For square belts:



$$\frac{T_1}{T_2} = e^{\mu_s \alpha}$$

Where μ_s is the static coefficient of friction, and α the wrap angle.

For v-shaped belts:



$$\frac{T_1}{T_2} = e^{\frac{\mu_s \alpha}{sin\gamma}}$$

Where γ is the angle of the v-section.

Clutches

For uniform pressure:

$$F = \pi p(R_1^2 - R_2^2)$$

$$T = \frac{2}{3}\mu F\left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right)$$

For uniform wear:

$$T = \mu F\left(\frac{R_1 + R_2}{2}\right)$$

Inertia

A body rotates with moment of inertia, I_G about its centre of gravity, G:

$I_G = \frac{1}{12}m(b^2 + h^2)$	$I_G = \frac{1}{2}mr^2$	$I_G = \frac{1}{12}mL^2$	$I_G = \frac{2}{5}mr^2$
Rectangular Laminar	Circular Laminar	Uniform Rod	Sphere

A body with radius of gyration k has moment of inertia: $I = mk^2$

Sums of Moments $\sum M = I\alpha$ $\sum M_P = -\bar{y}m(a_G)_x + \bar{x}m(a_G)_y + I_G\alpha$

Energy

The total kinetic energy, T, of a body is the sum of linear and rotational energies:

$$T = \left[\frac{1}{2}mv^2\right] + \left[\frac{1}{2}I_0\omega^2\right]$$

Impulse & Momentum

Impulse is the change in linear momentum:

$$\int_{t_1}^{t_2} \sum F\, dt = m(v_2 - v_1) = \Delta p$$

Angular Momentum at a point P relative to G:

$$H_P = I_G\omega + mv_x(y_G - y_P) - mv_y(x_G - x_P)$$

Angular Impulse:

$$\int_{t_1}^{t_2} \sum M_G\, dt = \int_{t_1}^{t_2} Fr\, dt = I_G(\omega_2 - \omega_1) = \Delta H_G$$